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## COMPARISON OF CAPACITORS BY MEANS OF A DOUBLE-CHANNEL DIGITIZER

**Abstract-** In the paper, a comparative method of measurement of the capacitors by measuring the voltages across them and phase angle between is presented. In the analysis of errors the systematic loading effects of the connection and their elimination has been described. The reductions of the systematic and the random errors of sampling and quantization have been performed in the frequency domain. Application of the method is possible with support of the modern digital signal processors and with interpolated DFT to improve the accuracy and speed of the measurement.

**Keywords:** measurement of capacitors, comparative method, error analysis, loading effect, leakage effect

### 1. INTRODUCTION

In measurement practice and especially in the industrial environment, comparative methods on a series of the measurement objects are frequently used. The parameters of the measurement objects are compared with the parameters of the reference one. This could be a reference etalon or it could be taken from the series of the objects and the suitable measurand is precisely measured. Because the values of the particular quantities are almost the same in the comparative measurements, the systematic effects of influence quantities can be well reduced [1], [2].

The basic electrical parameters of capacitors are the capacitance  $C$  and the quality of a dielectric expressed in terms of its 'loss tangent' or dissipation factor

$$d = \tan\delta = \frac{P}{Q} = \frac{U_R I}{U_C I} = \omega RC, \quad (1)$$

where  $R$  is an associated resistance in series with capacitance  $C$ . In the complete equivalent scheme we must take into consideration also the associated inductance  $L$  and the impedances of the terminals [3]. All mentioned quantities are frequency dependent and particularly  $R$  and  $\tan\delta$  are closely related with the analysis of the frequency characteristic of the impedance  $\underline{Z}$  which represents the unknown capacitor.

In most cases of the estimation of the frequency characteristics the bridge methods of measurement are used, where the active AC bridges [4], [5] and their derivation [6] come to the front. The measurements of the amplitude and the phase characteristics of the capacitors with the 'method of two voltmeters' are based on the measurement of the voltages across two almost equal capacitors with the intention of using the benefits of the comparative method also in the AC circumstances (Fig. 1).

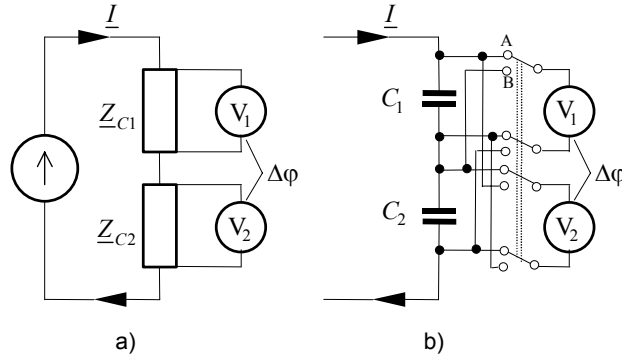


Fig. 1. An equivalent block scheme of the method of two voltmeters.

The method is derived from the 'method of three voltmeters', where the voltage (phasor) over both impedances is replaced with the phase angle between voltages across each impedance.

## 2. SYSTEMATIC ERRORS OF LOADING AND THEIR CORRECTION

In the serial equivalent scheme for a capacitor, the amplitude part of impedance can be written as:

$$Z_C = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \approx \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}. \quad (2)$$

In the approximation, an excitation frequency lower than  $f < 1/2\pi\sqrt{1/LC}$  is considered (for example:  $L < 10\text{nH}$ ,  $C < 100\mu\text{F}$ ,  $f < 160\text{kHz}$ ).

The ratio of the amplitude values of the voltages across two capacitors with the same sinusoidal excitation current is inversely proportional to the ratio of the capacitances if we consider a value for loss tangent lower than  $10^{-2}$  ( $\tan\delta = \omega RC \ll 1 \Rightarrow R \ll 1/\omega C$ ).

$$r = \frac{U_1}{U_2} = \frac{IZ_{C1}}{IZ_{C2}} = \frac{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_2}\right)^2}} \approx \frac{C_2}{C_1}. \quad (3)$$

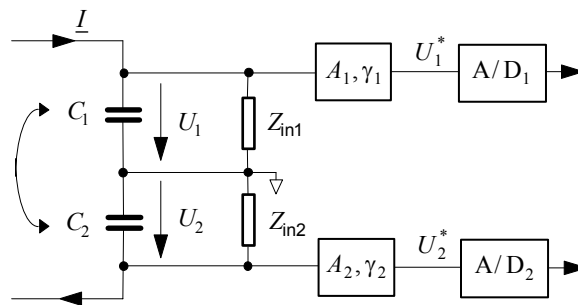


Fig. 2. An equivalent scheme of the inputs.

The voltmeters, which have been realized with the two-channel A/D converter (BB: DSP102) and the suitable front signal conditioning stages, have different gains ( $A_1, A_2$ ). They also have different input impedances ( $Z_{in1}, Z_{in2}$ ) including stray impedances (capacitances) of connection terminals to common ground (Fig. 2).

Voltages  $U_1^*$ ,  $U_2^*$  at inputs of A/D converters are systematically distorted by the loading effects of connection

$$\begin{aligned}\delta_1 &= -Z_{C1}/(Z_{C1} + Z_{in1}) \approx -Z_{C2}/(Z_{C2} + Z_{in1}); \\ \delta_2 &= -Z_{C1}/(Z_{C1} + Z_{in2}) \approx -Z_{C2}/(Z_{C2} + Z_{in2}).\end{aligned}$$

Input voltages  $U_1, U_2$  are also differently amplified. The gains of the input buffers, the nominal amplification factors, and the gains of A/D converters are included in the gain factors  $A_1$  and  $A_2$ . For voltages  $U_1^*$  and  $U_2^*$ , it can be easily deduced that

$$U_1^* = A_1 I Z_1 (1 + \delta_1) \quad U_2^* = A_2 I Z_2 (1 + \delta_2). \quad (4)$$

Because the capacitances  $C_1$  and  $C_2$  are close in value and considering that the input impedances are also very close, if we construct the input stages with equal components ( $\delta_1 \approx \delta_2$ ), we could satisfactory eliminate the systematic errors with the ratio of voltages.

$$r_A = \frac{U_{1A}^*}{U_{2A}^*} = \frac{A_1 Z_1 (1 + \delta_1)}{A_2 Z_2 (1 + \delta_2)} \approx \frac{A_1 (1 + \delta_1) C_2}{A_2 (1 + \delta_2) C_1} = \frac{A_1 (1 + \delta_1)}{A_2 (1 + \delta_2)} r_C. \quad (5)$$

If we want to eliminate the different gain factors  $A_1$  and  $A_2$ , we change the positions of the capacitors (switch position A to B of the commutator on Fig. 1b).

$$r_B = \frac{U_{1B}^*}{U_{2B}^*} = \frac{A_1 Z_2 (1 + \delta_1)}{A_2 Z_1 (1 + \delta_2)} \approx \frac{A_1 (1 + \delta_1) C_1}{A_2 (1 + \delta_2) C_2} = \frac{A_1 (1 + \delta_1)}{A_2 (1 + \delta_2)} \frac{1}{r_C}. \quad (6)$$

We can deduce the desired ratio of two capacitances from (5) and (6), if their values are close enough ( $|C_1 - C_2|/C_1 < 10^{-2}$ ).

$$\frac{r_A}{r_B} = r_C^2 \quad \Rightarrow \quad \frac{C_2}{C_1} = \sqrt{\frac{r_A}{r_B}}. \quad (7)$$

Like amplitude, the phase angle is also distorted. The problem of estimation of the difference of the loss tangents ( $\tan \delta_1 = 1/\tan \varphi_1 \ll 1$ ,  $\tan \delta_2 = 1/\tan \varphi_2 \ll 1$ ;  $\varphi_1 = \angle(\underline{I}, \underline{U}_1)$ ,  $\varphi_2 = \angle(\underline{I}, \underline{U}_2)$ ) can be translated into the problem of estimation of a small phase difference between sinus signals ( $\Delta \delta = \delta_1 - \delta_2 = \varphi_2 - \varphi_1 = -\Delta \varphi$ ).

$$\tan(\delta_1 \pm \Delta \delta) \approx \tan(\delta_1) \pm \tan(\Delta \delta) \quad (8)$$

Systematic phase deformations ( $\gamma_1, \gamma_2$ ) could be eliminated with the subtraction in the scheme of changing the positions of the impedances.

$$\begin{aligned}\varphi_{1A}^* &= \varphi_1 + \gamma_1 & \varphi_{2A}^* &= \varphi_2 + \gamma_2 \\ \varphi_{1B}^* &= \varphi_2 + \gamma_1 & \varphi_{2B}^* &= \varphi_1 + \gamma_2\end{aligned}\quad (9)$$

The additional phase angles between voltages and current  $\gamma_1$  and  $\gamma_2$  are the contributions of the input stages and the A/D converters. After exchange of the capacitor positions the phase difference can be easily obtained as

$$\begin{aligned}\Delta\varphi_A^* &= \varphi_{1A}^* - \varphi_{2A}^* = \varphi_1 - \varphi_2 + \gamma_1 - \gamma_2 \\ \Delta\varphi_B^* &= \varphi_{1B}^* - \varphi_{2B}^* = \varphi_2 - \varphi_1 + \gamma_1 - \gamma_2\end{aligned},\quad (10)$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = \frac{\Delta\varphi_A^* - \Delta\varphi_B^*}{2} = -\Delta\delta.\quad (11)$$

### 3. SYSTEMATIC AND RANDOM ERRORS OF SAMPLING

The first step in the digital measurement procedure is uniform sampling of the time dependent signal. The sampled analog signal  $g(t)$  in our case sinusoidal with excitation frequency  $f_0$  and additional disturbing frequencies  $f_m$ , can be written as follows:

$$g(k\Delta t)_N = w(k)g(k\Delta t)_\infty = \sum_{m=0}^M A_m \sin(2\pi f_m k\Delta t + \varphi_m).\quad (12)$$

From the infinite line of sampling points only a finite number  $N$  ( $k = 0, 1, \dots, N - 1$ ) is taken with a multiplication of the theoretically infinite sampled signal with a window of finite duration  $w(k)$ .  $A_m, f_m$  and  $\varphi_m$  are the amplitude, frequency, and phase relevant to the measured component of the signal. The sampling frequency  $f_s = 1/\Delta t$  is supposed to fulfil the Shannon theorem  $f_s > 2f_M$  ( $f_M$ - the highest frequency component).

Continuing the measurement procedure, we perform DFT of the obtained sequence of samples. Multiplication of the sampled signal  $g(k\Delta t)$  by the finite duration window  $w(k)$  in the time domain causes convolution of the frequency transform of the signal (for the single component that is a Dirac impulse with amplitude  $A_m$  and phase  $\varphi_m$ ) with the frequency transform of the window  $W(\theta)$ . The DFT of the signal on  $N$  sampled points (12) at the spectral line  $i$  is given by [7]:

$$G(i) = -\frac{j}{2} \sum_{m=0}^M A_m [W(i - \theta_m) e^{j\varphi_m} - W(i + \theta_m) e^{-j\varphi_m}].\quad (13)$$

$\theta_m$  is the component frequency related to base frequency resolution depending on the window span  $\Delta f = 1/N\Delta t$ . Spectral lines of the signal do not coincide with the DFT coefficients in general, unless the measurement interval  $N\Delta t$  is an integer multiple of the period of the frequency signal component  $T_m = 1/f_m$ . For this reason the relative frequency can be written in two parts:

$$\theta_m = \frac{f_m}{\Delta f} = i_m + x_m ; \quad -0.5 < x_m < 0.5, \quad (14)$$

$i_m$  is an integer value, determining approximately the position of the signal component. The displacement term  $x_m$  is caused by lack of synchronization.

For estimation of displacement  $x_m$  of a single component, at first, we idealize circumstances and in the Eq. (13) neglect the second part and contributions of other signal components. This means in practice that the measurement lasts long enough, or, the frequency components are sufficiently interspaced [8]. For a two-point interpolation the local maximum in the amplitude part of DFT with the largest coefficients  $|G(i)|$  and  $|G(i+1)|$ , surrounding the position of component  $m$ , must be found. Considering the Eq. (14), we can write as follows:

$$\alpha_m = \frac{|G(i_m \pm 1)|}{|G(i_m)|} = \frac{|W(1-x_m)|}{|W(x_m)|} \Rightarrow x_m = f(\alpha_m). \quad (15)$$

The component amplitude is obtained if the function  $W(\theta)$  of the window used is analytically known:

- Rectangular window:  $A_m = \frac{2}{N} |G_m(i)| \frac{\pi x_m}{|\sin(\pi x_m)|}$ .
- Hanning window:  $A_m = \frac{2}{N} |G_m(i)| \frac{\pi x_m (1-x_m^2)}{|\sin(\pi x_m)|}$ . (16)

For the component phase can be written [10]:

$$\varphi_m = \arg G_m(i) - \pi \frac{N-1}{N} x_m + \frac{\pi}{2}. \quad (17)$$

The nonsynchronization of the measurement interval  $T = N\Delta t$  with a multiple of the period of the frequency signal component  $T_m$ , that exhibits as shifting of the component frequency  $f_m$  from the largest local amplitude coefficient of the DFT, cause the short-range and the long-range leakage influences on the DFT coefficients. The short-range leakage effects on the amplitude and phase estimation can be well corrected by interpolations as in (16) and (17). The long-range leakage effects are reduced by a suitable window and the multipoint interpolation [9]. Both tasks could be very difficult if an analytically not well-defined window is used.

The leakage effects can be very much reduced with a comparative measurement, if the simultaneousness of the sampling on both channels is assumed and the measurement time of signals is the same. The assurance of these conditions gives equal displacements  $x_1$  and  $x_2$ , if the measurement frequency is the same.

The comparative method easily eliminates the displacement contributions in (16) and (17), if  $A_{m1} = U_1^*$  and  $A_{m2} = U_2^*$  are considered.

$$r = \frac{U_1^*}{U_2^*} \approx \frac{|G_1(i)|}{|G_2(i)|}; \quad \varphi_1 - \varphi_2 \approx \arg G_1(i) - \arg G_2(i). \quad (18)$$

It is reasonable to take the amplitude and the phase of the largest local DFT coefficient, where the signal to noise ratio is largest. The quantization error is a minimum that must be taken into consideration in the measurement uncertainty of the final result. The error is assumed to be random with the uniform density distribution between  $-\Delta/2$  and  $\Delta/2$  ( $\Delta = G_{\text{Range}}/(2^n - 1)$ ) in the time space and with the 'white' spectrum in the frequency space. The probability density distribution of DFT coefficients is Gaussian owing to the great number of summations in DFT procedure. For the rectangular window, the SNR is given by [11]:

$$\text{SNR}_{\text{DFT}}(i) = \frac{A}{\Delta} \sqrt{\frac{12N}{\pi}} \quad \text{or} \quad \text{SNR}_{\text{DFT}}(i) = \frac{A}{\Delta} \sqrt{\frac{12N}{\pi \text{ENBW}}}, \quad (19)$$

for an arbitrary window. The  $S/N$  ratio is improved with the square root of the number of points at given quantization and it decreases with the equivalent noise bandwidth ( $\text{ENBW}$ ). The least one has a rectangular window ( $\text{ENBW} = 1$ ). In our case the Hanning window with larger noise bandwidth ( $\text{ENBW} = 1.5$ ) has been used, but the systematic leakage errors are smaller.

In spite of good correction properties of the comparative method, the measurement time must be close to the multiple of the component period. Even more important, the measurement time should be a multiple of the period of the difference between excitation frequency and disturbing ones (In most cases 50/60 Hz in the supply systems.  $f_m = if_{50}$ ,  $i = 1, 2, \dots$  multiple of the supply frequency).

$$T = N\Delta t \approx p \frac{1}{f_0} \approx r \frac{1}{|f_0 - f_{50}|}. \quad (20)$$

For the Hanning window the constants must be  $p, r \geq 3, 4, \dots$  ( $p = r(2i+1)$ ). The excitation frequency should be in the middle of the two successive multiples of the supply frequency ( $f_0 = (i + 1/2)f_{50}$ ). The measurement time is given by:

$$T = \frac{p}{(i + 1/2)f_{50}} = \frac{r(2i + 1)}{i + 1/2} T_{50}. \quad (21)$$

#### 4. EXPERIMENTAL RESULTS

To check the proposed method a laboratory prototype was constructed. A two-channel A/D converter ( $f_s < 200$  kHz) with the resolution of 13 effective bits has been used. For signal processing the DSP module with TMS 320C30 has been added.

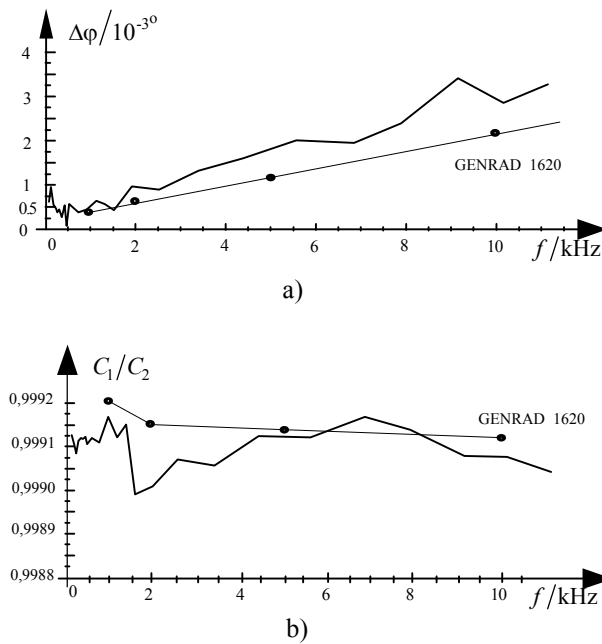


Fig. 3. Phase difference  $\Delta\varphi$  (a) and the ratio of capacitances  $C_1/C_2$  (b)  
 ( $C_1 = 180,78\text{nF}$  ;  $C_2 = 180,92\text{nF}$ ).

The distribution of  $\Delta\varphi$  measurements has been near the level of the theoretical 12-bit A/D converter ( $s_{(15\text{measurements})} = 0.6 \times 10^{-3^\circ}$ ). The results have been compared (Fig. 3.) with the reference measurement instrument GENRAD 1620 (measurement accuracy:  $ma_C = 10^{-4}$  ;  $ma_{\tan\delta} = 5 \times 10^{-4}$  ;  $ma_\varphi = 1.7 \times 10^{-3^\circ}$ ) and confirm the success of the method and the possibility of its use in an industrial environment.

## 5. CONCLUSIONS

A comparative method - method of two voltmeters - of measurement of the capacitors by measuring the voltages across them and phase angle between has been presented to show the effectiveness of the measurement system. In the analysis of errors the systematic loading effects of the connection and their elimination have been described.

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## POMIARY POJEMNOŚCI ZA POMOCĄ DWUKANAŁOWEGO KOMPARATORA CYFROWEGO

### Streszczenie

W pracy przedstawiono komparacyjną metodę pomiaru pojemności kondensatorów drogą pomiaru napięć i różnicy fazy. W analizie dokładności uwzględniono błędy wynikające z połączeń i podano sposób ich eliminacji. Metodą cyfrowego przetwarzania sygnałów w dziedzinie częstotliwości osiągnięto redukcję systematycznych i przypadkowych błędów próbkowania i kwantyzacji.